Introduction to Probability

MA 421–Spring 2001
MWF 12:25-1:15 pm–HA 366

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Course Description and Goals

Probability surrounds us; from lotteries to space shuttle launches, from the validity of disease tests to criminal court cases. In this introductory probability course, you will determine the odds and probabilities of various games of chance, will learn the common distributions used to model real-world situations and will appreciate and assess the use of probability in the world today. Some unique features of this course are Casino Week, paper assignments and a group project. During Casino Week, you will gamble with extra credit points by "playing" games of chance (that is, solving problems of chance). Several papers on applications of probability will be due throughout the semester. They are designed to increase your excitement for the infinite uses of probability. A group project will be due at the end of the semester. Here your group gets an opportunity to use probability to tackle a problem of your choice and present the results to the class.

As a graduate of this course you will be able to do the following:

- prove probabilistic laws and statements.
- explain and define fundamental concepts of probability.
- compute probabilities, expected values and variances.
- extend the fundamental concepts of probability to model problems and questions in the real world probabilistically.
- appreciate probability's use in the world around you and assess the role of probability in the public domain.

Prerequisites

Students are expected to be well-versed in differential and integral calculus. The final section of this class will also require some knowledge of multiple integration. Since this is an upper-level mathematics course, some "mathematical maturity" will be expected. This means you will be required to write some elementary proofs and understand the theoretical underpinnings of some statements. MA 225 has helped many students achieve this "mathematical maturity" but is not required. If you feel your background is lacking at any time during the semester and are willing to put in extra time, please see me to arrange extra help sessions.

Text

Required Text: Ross, Sheldon M. *A First Course in Probability.* Fifth ed. New Jersey: Prentice Hall, 1998.

Suggested References:

- Ross, Sheldon M. *Introduction to Probability Models.* Fifth ed. San Diego, CA: Academic Press, Inc., 1993.
- Snell, J. Laurie and Charles M. Grinstead. *Introduction to Probability.* Second ed. Providence, RI: American Mathematical Society, 1997.
- Websites: Chance website: `http.://www.dartmouth.edu/~chance/`

Course Requirements and Evaluation

Students will be evaluated by numerous homework assignments, papers and projects. In addition, there will be two in-class exams and one take-home final. The homework assignments will occur about every 10 days. They will generally consist of 5-8 well-chosen problems which will allow you to practice the material and will allow me to assess the progress of the class. These problems will address goals 1 and 3 of this course. That is, the problems will include both proofs and computational exercises. The paper assignments and the Casino Week assignment address course goals 4 and 5. Hopefully, in completing the papers, you will develop an appreciation for the uses and modeling complexities of probability. The final group project will require a preliminary proposal, outline, progress meeting, final written report and oral in-class presentation. More details on the more involved assignments, such as the paper submissions and group project will be supplied at the appropriate times during the semester. (The Casino Week assignment and one paper assignment are included at the end to give you an idea of these paper assignments and my expectations.)

Clearly, the setup of the evaluation is such that memorization of material for in-class examination is not emphasized. Rather, the evaluation is designed to help students develop proof-writing, computational, and modeling skills in addition to communication skills. Evaluation is formative rather than summative, meaning I encourage you to view each assignment as a learning *process*; drafting, discussing during office hours, reviewing feedback, revising, submitting and resubmitting are encouraged. You need not worry that your initial submission will be your final submission, upon which I place a final judgment as to your ability. Partial credit is given for valiant attempts at homework problems and all papers and projects are graded based on your achievement of a predetermined set of criteria, which are included with each assignment. This course will require a great deal of work which will be spread out throughout the semester. The number of assignments and their frequency are designed to force you to learn continually and to avoid cramming before tests. I have also designed the assignments with a great deal of choice and variety in order to encourage creativity and enjoyment of the subject.

The weighting of all evaluation instruments is given below:

- Homeworks (6) – 18%
- Casino Week – 5%
- In-class exams – 20%
- Papers (6) – 27%
- Group Project – 15%
- Take-home final – 15%

The University plus/minus scale for grading will be used to assign a final letter grade for the course.

Examples of Assignments

To give you an idea of the nature of the assignments in this class and my expectations, I have included two assignments you will see during the semester. One pertains to Casino Week, the week where you gamble with extra credit points by "playing" games of chance (ie, solving problems of chance). The second assignment is a typical paper assignment, requiring you to relate the concept of conditional probability to disease testing.

Course Policies

Class attendance is not mandatory. Yet attendance does correlate with success in any class. During many classes we will do group work or discuss problems which cannot be found in the book. Thus attending class will be to your benefit. Late assignments will not be accepted, except in extenuating circumstances (death in the family, university activity requiring travel). When possible, make arrangements prior to your absence and submit your work early. Plagiarism will not be accepted! If you are ever unsure as to what constitutes plagiarism, see me.

Course Organization

Combinatorial Analysis (Ch. 1)

Week 1

- M, Jan. 8 – course introduction, *Work Assigned:* Read Ch. 1.
- W, Jan. 10 – 1.1-1.3
- F, Jan. 12 – 1.4-1.5, concept map
  - use Venn diagrams to begin proofs of simple probability laws from chapters 1.

  - know the difference between factorial, permutation, combination and multinomial coefficients and recognize when to use which one.
  - use the basic principles of counting to compute the number of ways of arranging items.
  - determine the number of outcomes in a sample space.

Axioms of Probability (Ch. 2)

Week 2

- M, Jan. 15 – MLK Holiday
- W, Jan. 10 – 2.1-2.2, *Work Assigned:* HW #1, concept map Ch. 2
- F, Jan. 12 – 2.3
  - state the axioms of probability and build proofs from these first principles.
  - state DeMorgan's laws and explain the importance and use of these laws. Be able to prove one statement of DeMorgan's laws.
  - summarize in your own words the following terms: sample space, event, outcome, equally likely, probability, complement, union, intersection.
  - explain the alternate (and more intuitive) view of probability as a relative frequency.
  - compute probabilities for a sample space with equally likely outcomes.

Week 3

- M, Jan. 22 – 2.4, proof practice, *Work Assigned:* Casino Week
- W, Jan. 24 – 2.5, *Work Due:* initial Casino Part 1
- F, Jan. 26 – practice problems, hw day *Work Due:* HW #1
  - be familiar with the complementary view of a problem.
  - recognize when a sample space has equally likely outcomes.
  - use complementarity to make probabilities easier to compute.

Week 4

- M, Jan. 29 – Casino Week, class meets, *Work Due:* revised Casino Part 1
- W, Jan. 31 – Casino Week, optional class attendance
- F, Feb. 2 – Casino Week, class meets, present tough problems, *Work Due:* Casino Week portfolio
  - compute probabilities associated with games of chance and use expected value to determine which games of chance are best to play.

Conditional Probability and Independence (Ch. 3)

Week 5

- M, Feb. 5 – 3.1-3.2, *Work Assigned:* HW #2
- W, Feb. 7 – 3.3, *Work Assigned:* Odds Paper
- F, Feb. 9 – 3.4, *Work Assigned:* Concept Map Ch. 3
  - state the formal definition of conditional probability and the intuitive concept behind conditional probability.
  - explain the concept of odds and the odds ratio, present an argument for why odds is more popular in the public domain.
  - state the multiplication rule and Bayes' theorem.
  - explain the concept of odds and the odds ratio, present an argument for why odds is more popular in the public domain.

Week 6

- M, Feb. 12 – practice problems, *Work Due:* Concept Map Ch. 3
- W, Feb. 14 – HW day, review, *Work Due:* HW #2
- F, Feb. 16 – In-class Exam
  - explain independence and understand its importance.
  - use the multiplication rule and Bayes' Theorem to compute conditional probabilities in problems in which extra information is known.
  - determine the probability that you have a disease, given that you have tested positive for the disease twice.

Random Variable (Ch.4)

Week 7

- M, Feb. 19 – 4.1, *Work Due:* Odds Paper, *Work Assigned:* Proof Paper
- W, Feb. 21 – 4.2, *Work Due:* Disease Paper
- F, Feb. 23 – 4.3,4.4
  - prove the expected value and variance for one discrete distribution using the definitions of expected value and variance.
  - define random variable, range of a random variable, expected value, variance, probability distribution function, and cumulative distribution function.
  - understand the concepts of expected value as a weighted average, variance and standard deviation as measures of spread and expectation as a linear operator.

Week 8

- M, Feb. 26 – 4.5-4.6, *Work Due:* Proof Paper, *Work Assigned:* HW #3
- W, Feb. 28 – 4.7, *Work Assigned:* Concept Map Ch. 4
- F, Mar. 2 – 4.8, *Work Assigned:* Uses of Discrete R.V.
  - assess the value of proof in mathematics.
  - understand when and how to use approximations to distributions.

Week 9

- M, Mar. 5 – 4.9, *Work Due:* Concept Map Ch. 4, *Work Assigned:* Project proposal HW #4
- W, Mar. 7 – practice problems, *Work Due:* Uses of Discrete R.V.
- F, Mar. 9 – HW day, brainstorm, *Work Assigned:* HW #3
  - solve problems based on the common discrete distributions.
  - brainstorm uses of each of the discrete random variables presented.
  - formulate good problem questions and proposals.

Continuous Random Variables (Ch. 5)

Week 10 – No Class – Spring Break

Week 11

- M, Mar. 19 – 5.1, 5.7, *Work Assigned:* Chance paper
- W, Mar. 21 – 5.7, *Work Assigned:* Concept Map Ch. 5
- F, Mar. 23 – 5.2, *Work Assigned:* HW #4
  - prove the expected value and variance for one continuous distribution using the definitions of expected value and variance.
  - compute the distribution of a function of a random variable.
  - assess the appropriateness of statistical measures used in the public domain and discussed in one *Chance* article of choice.

Week 12

- M, Mar. 26 – 5.3
- W, Mar. 28 – 5.4, *Work Due:* Chance paper *Work Assigned:* Uses of Cont. R.V.
- F, Mar. 30 – 5.4, *Work Due:* Concept Map Ch. 5

  - identify the differences in computing expected value and variance for discrete versus continuous random variables.
  - list the common continuous random variables (uniform, normal, exponential) and their probability distribution functions along with the assumptions and uses of each random variable. Know where to find the already proven formulas for the expected value and variance of each distribution.
  - understand when and how to use approximations to distributions.

Week 13

- M, Apr. 2 – 5.5, *Work Due:* Project proposal
- W, Apr. 4 – practice problems, *Work Due:* Uses of Cont. R.V.
- F, Apr. 6 – HW day, *Work Due:* HW #4, *Work Assigned:* Project Outline
  - compute probabilities for the normal distribution by converting to the standard normal distribution with a z-score and using the table.
  - use approximations to compute probabilities and expected values for distributions which involve tedious, lengthy computations.
  - list the two memoryless distributions and explain for which types of problems these are popular modeling tools and why.
  - design projects which can be solved with probability and evaluate the limitations of probability in solving these problems.

Jointly Distributed Random Variables (Ch. 6) and Properties of Expectation (Ch.7)

Week 14

- M, Apr. 9 – 7.1-7.2, *Work Assigned:* HW #5, Concept Map Ch. 7
- W, Apr. 11 – No Class
- F, Apr. 13 – No Class
  - define and explain the concept of expected value of sums of random variables.
  - compute the expected value of sums of random variables.

Week 15

- M, Apr. 16 – In-class Exam #2, 7.6 *Work Due:* Project Outline
- W, Apr. 18 – 7.4 *Work Due:* Concept Map Ch. 7
- F, Apr. 20 – HW day, practice problems, *Work Due:* HW #5
  - state the law of total probability.
  - define and explain the concepts of expected value of sums of random variables, mgf.

- know where to find the mgf's for common distributions.
- explain the use of mgf.
- apply the law of total probability to compute otherwise difficult expected values and probabilities.
- use mgf to calculate the expected value, variance, pdf of one common distribution.

Week 16

- M, Apr. 23 – 6.1, *Work Due:* Project Draft (optional)
- W, Apr. 15 – 6.2,
- F, Apr. 27 – 6.3,
  - prove one statement involving sums of random variables (i.e., prove that the sum of binomial random variables with parameters (n,p) and (m,p) is binomially distributed with parameters (n+m,p).)
  - compute probabilities and expected values of jointly distributed random variables.

Week 17

- M, Apr. 30 – projects
- W, May 2 – projects
- F, May 4 – projects, *Work Assigned:* Take-home final
  - complete and present a team project which solves an approved probability problem analytically, by simulation or as an experiment with detailed assumptions.

FINAL–M, May 7, 1-4 pm